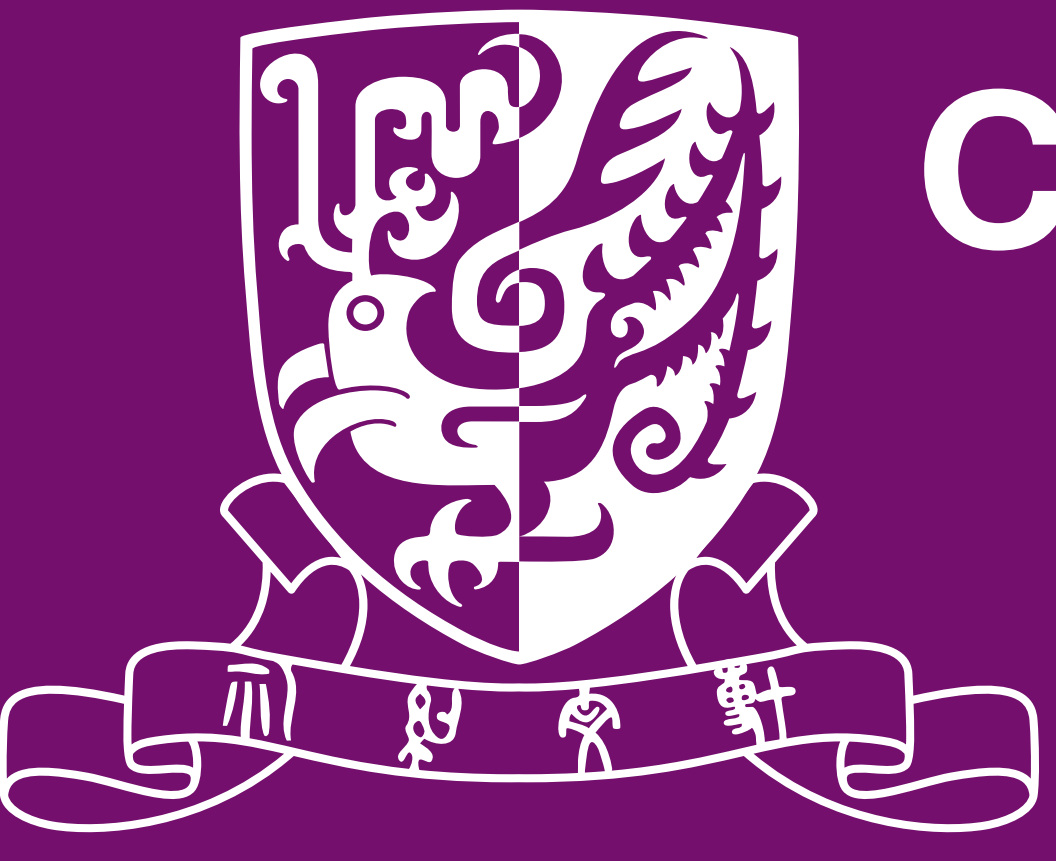




Canonical Quantum Mpemba Effect in a Dissipative Qubit

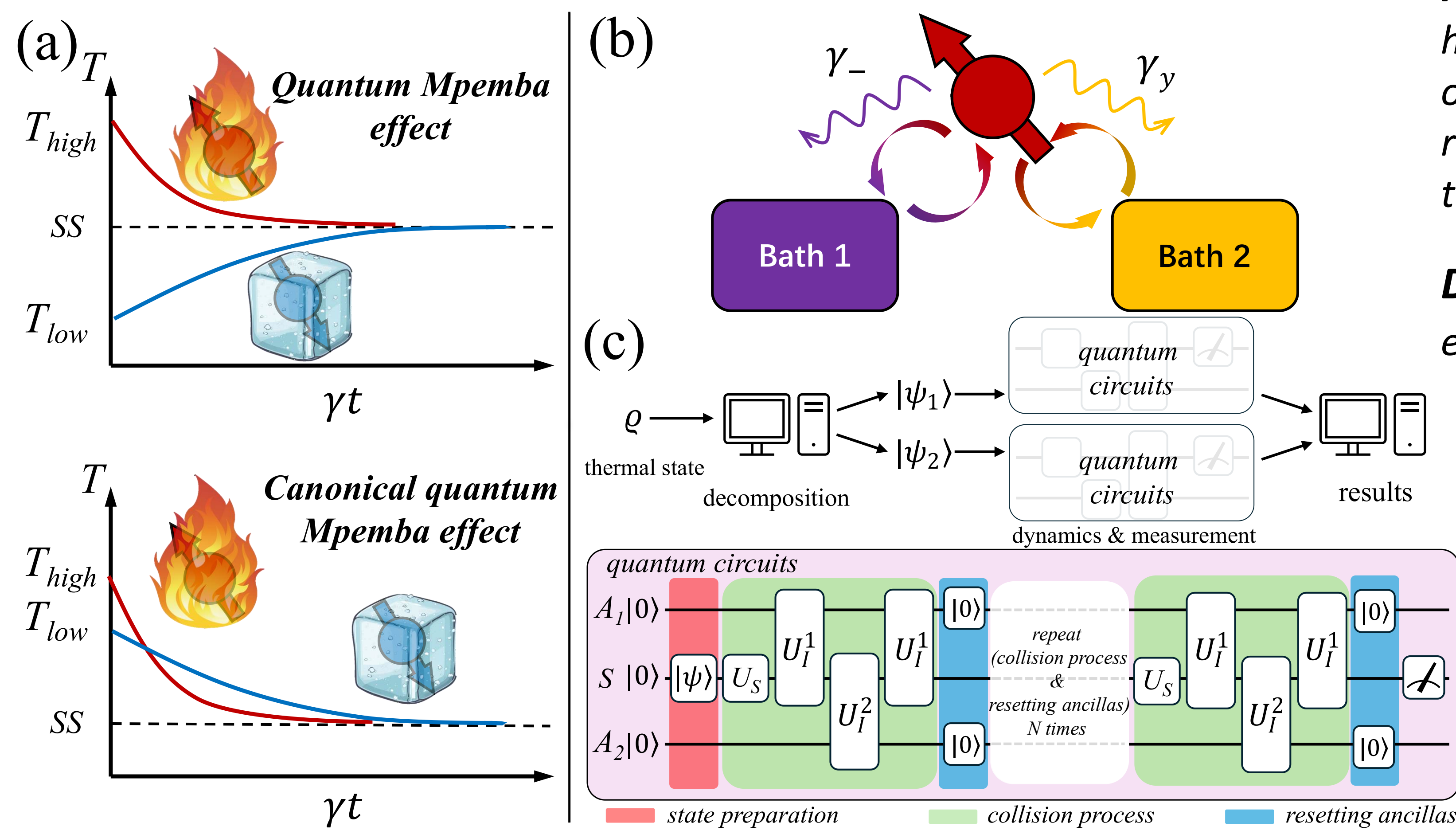
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Abstract

The Mpemba effect, where a hotter system cools faster than a colder one under otherwise identical conditions, has been extensively studied in classical systems. In this work, we present the quantum analogue of the Mpemba effect using a dissipative qubit. We demonstrate that, under the identical conditions, the relaxation dynamics of a qubit initialized in a thermal state with a higher temperature can be exponentially faster than that of a colder thermal state. Strikingly, this acceleration is determined solely by the initial temperature of the system, independent of other parameters. By defining the effective temperature for the steady state, we confirm that the relaxation process constitutes a genuine cooling process, analogous to its classical counterpart. Moreover, we propose a practical quantum circuit to realize this effect experimentally using superconducting qubits.

Model



Motivation: In open quantum systems, the Mpemba effect is the phenomenon where a hotter initial state leads to more rapid relaxation to the steady state compared to a colder one. However, if the steady-state temperature lies between them, their markedly different relaxation processes (heating vs. cooling) deviate from the classical Mpemba effect. Thus, the steady state's effective temperature must be accounted for.

Definition: The temperature satisfies $T_{\text{high}} > T_{\text{low}} > T_{\text{SS}}$, with T_{SS} denotes the steady-state effective temperature, while the critical times to reach steady state satisfying $t_c^{\text{high}} < t_c^{\text{low}}$.

Lindblad master equation
$$\frac{d}{dt} \varrho = -i[H_S, \varrho] + \gamma_- D_{\varrho}[\sigma_-] + \gamma_y D_{\varrho}[\sigma_y],$$

Time-evolution dynamics
$$\varrho(t) = e^{t\mathcal{L}}[\varrho_{\text{ini}}] = \varrho_{\text{SS}} + \sum_{k=2}^d \text{Tr}[l_k \varrho_{\text{ini}}] r_k e^{\lambda_k t},$$

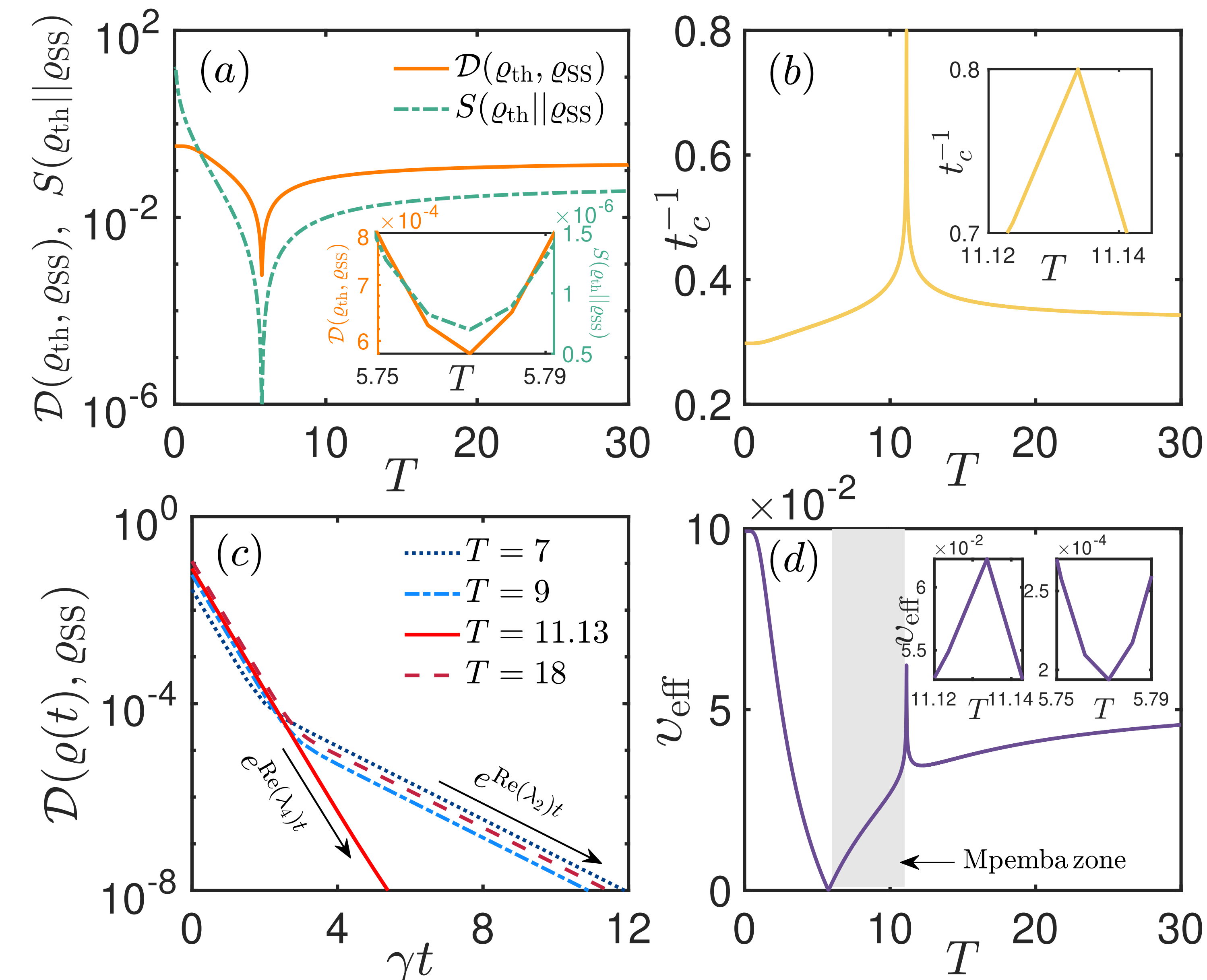
The steady-state effective temperature

$$T_{\text{SS}} = \arg \min_T \{D(\varrho_{\text{SS}}, \varrho_{\text{th}}[T])\},$$

The time-evolved trace distance

$$D(\varrho(t), \varrho_{\text{ss}}) = \frac{1}{2} \text{Tr}[\sqrt{(\varrho(t) - \varrho_{\text{ss}})^\dagger (\varrho(t) - \varrho_{\text{ss}})}].$$

Results

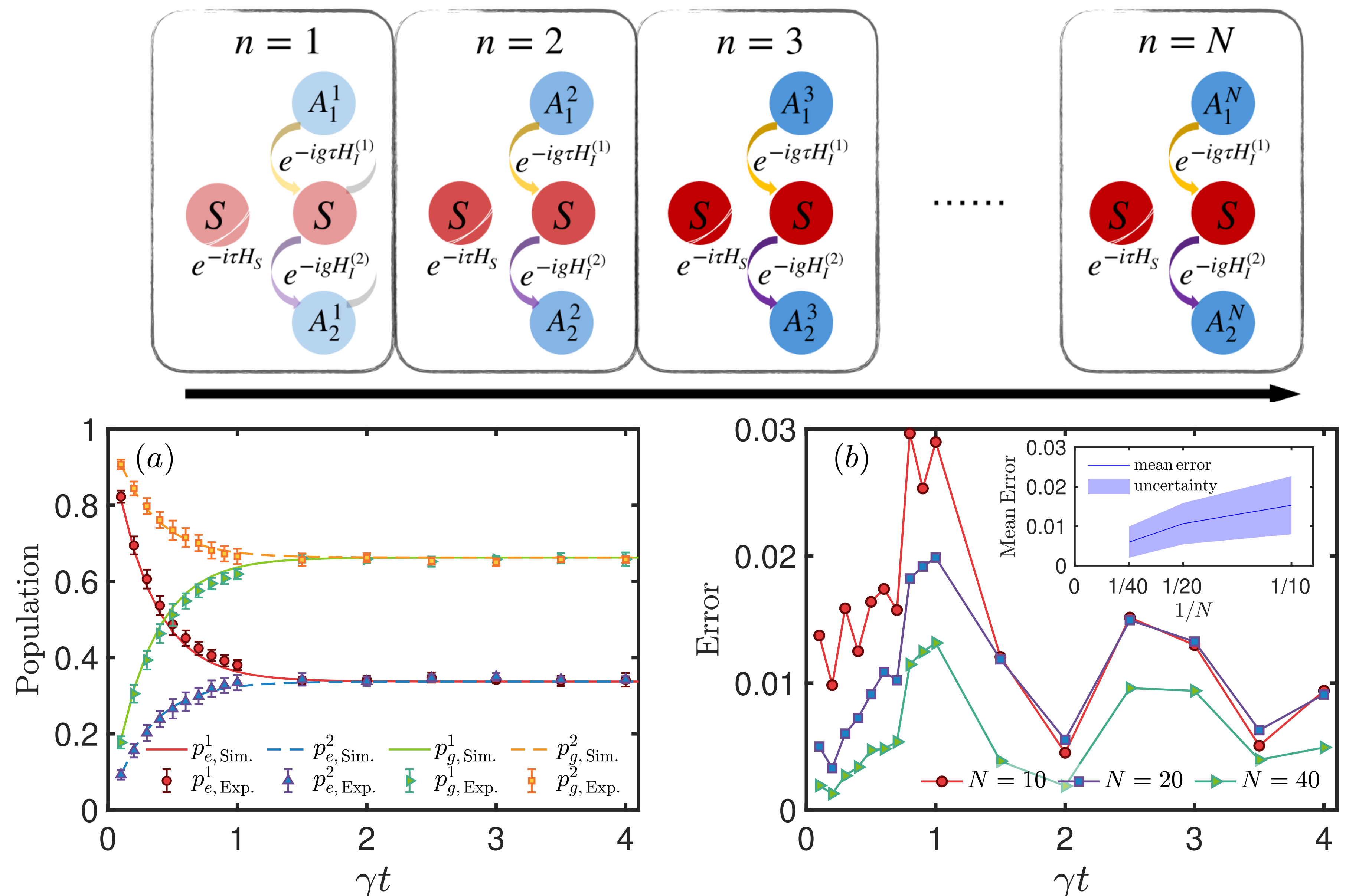


Numerical Simulation

- (a) Trace distance (orange solid line) and quantum relative entropy (green solid line) between the steady state and the thermal states indicate that the steady-state effective temperature is about $T \approx 5.77$.
- (b) The inverse critical time t_c^{-1} to reach the steady state as a function of the temperature, showing that when the temperature of initial thermal state is $T \approx 11.13$, the system will reach steady state more quickly.
- (c) We demonstrate the trace distance between the thermal states at four different initial temperatures and the steady state during the relaxation process, illustrating the thermal Mpemba effect.
- (d) In the Mpemba zone, the thermal state has a higher initial temperature will reach steady state more quickly.

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Experimental Implementation

Quantum collision model: The environment is simulated as a collection of ancillas. The system's interactions with its environment is discretized into a sequence of pairwise unitary interactions ("collisions") between the system and individual ancillas. The Lindblad master equation thus can be decomposed into elementary collision steps:

$$\varrho_S^n \mapsto \varrho_S^{n+1} = \text{tr}_{\eta_A^{n+1}} [U_I U_S \varrho_S^n \otimes \eta_A^{n+1} U_S^\dagger U_I^\dagger]$$

- (a) Comparison of the experimental data (markers) obtained by the superconducting qubits and theoretical results (lines). Here we use a classical-quantum hybrid algorithm to simulate the dissipation dynamics of the thermal state.
- (b) The errors can be suppressed by performing statistical analysis on repeated experiments.

