Dynamical Phase Transition of Dissipative Fermionic Superfluids

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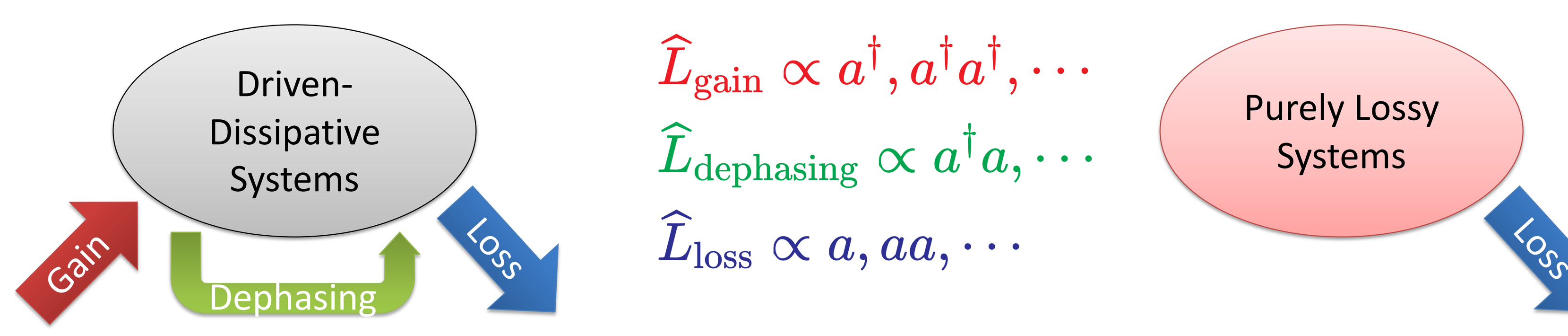
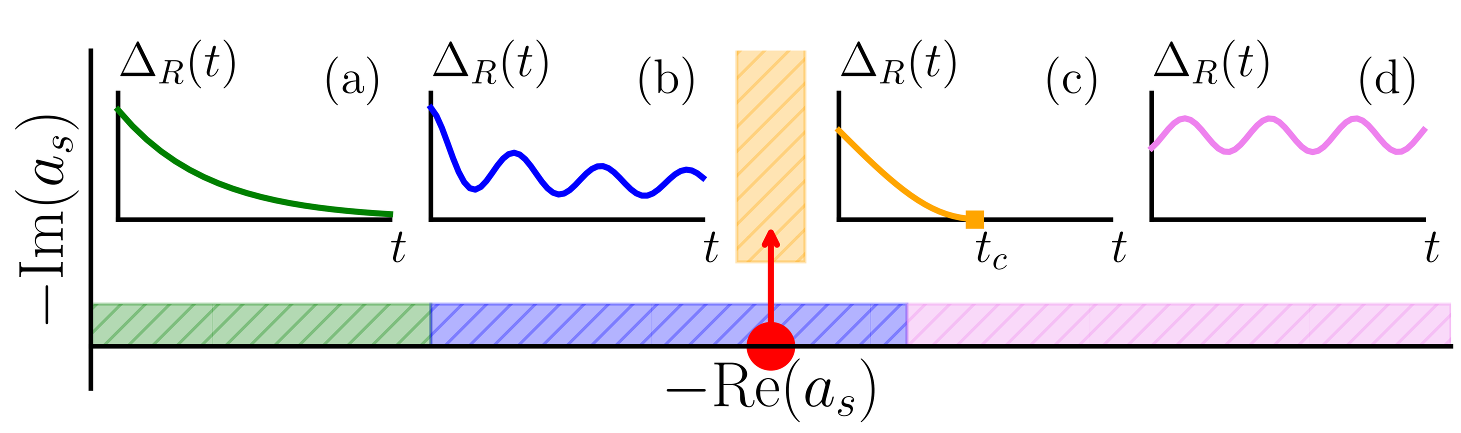
Introduction

The interplay between coherent evolution and dissipation gives rise to rich physical phenomena in driven-dissipative open quantum many-body systems. Much attention has been devoted to understanding non-equilibrium phases of different steady states. However, **purely lossy open quantum many-body systems**, such as ultracold atomic gases subject to one- or two-body losses, **inevitably evolve towards the vacuum state**, making steady-state analyses insufficient. For such systems, the transient dynamics preceding vacuum decay become the primary focus, demanding theoretical frameworks that can capture the full temporal evolution.



A particularly important example is **dissipative fermionic superfluids [i.e., fermionic superfluids with particle losses due to inelastic collision]**, where inelastic scattering introduces complex dynamics that fundamentally differ from their closed-system counterparts. While the dynamics of elastic scattering length quenches in superfluids are well-understood, producing oscillating or exponentially decaying order parameters [subfigures (a,b,d) below], the effects of inelastic scattering have received limited theoretical attention. Previous studies using Anderson's pseudo-spin formalism on lattices [1-3], assume a pure superfluid without accounting for the normal component that emerges during dissipative evolution. This gap motivates our investigation of homogeneous lossy fermionic superfluids in continuous space, where we discover a fundamentally **new type of "dynamical phase transition", where the superfluid closes its gap at a finite critical time** [subfigures (c) below].



Methodology

1. Model

- We consider a spin-balanced Fermi gas under a general external potential. The momentum-space Hamiltonian reads

$$\widehat{H} \equiv \widehat{H}_R = \sum_{\mathbf{q},\mathbf{q}',\sigma} [(\epsilon_{\mathbf{q}} - \mu)\delta_{\mathbf{q},\mathbf{q}'} + U(|\mathbf{q}-\mathbf{q}'|)] c^\dagger_{\mathbf{q},\sigma} c_{\mathbf{q}',\sigma} + \frac{g_R}{V} \sum_{\mathbf{P},\mathbf{q},\mathbf{q}'} c^\dagger_{\frac{\mathbf{P}}{2}-\mathbf{q}',\uparrow} c^\dagger_{\frac{\mathbf{P}}{2}+\mathbf{q}',\downarrow} c_{\frac{\mathbf{P}}{2}+\mathbf{q},\downarrow} c_{\frac{\mathbf{P}}{2}-\mathbf{q},\uparrow}$$

- We consider two possible channels for dissipation, i.e., one- and two-body losses

$$\widehat{L}_{1,\mathbf{k},\sigma} = \sqrt{2\gamma} c_{\mathbf{k},\sigma},\ \widehat{L}_{2,\mathbf{P}} = \sqrt{\frac{2g_I}{V}} \sum_{\mathbf{q}} c_{\frac{\mathbf{P}}{2}+\mathbf{q},\downarrow} c_{\frac{\mathbf{P}}{2}-\mathbf{q},\uparrow},\ \widehat{H}_I \equiv -\sum_n \widehat{L}^\dagger_n \widehat{L}_n/2.$$

- The exact evolution of the system is described by the Lindblad master equation

$$i\hbar \frac{d\widehat{D}}{dt} = [\widehat{H}, \widehat{D}] + i\hbar \sum_n \left(\frac{1}{2}\{\widehat{L}^\dagger_n \widehat{L}_n, \widehat{D}\} - \widehat{L}_n \widehat{D} \widehat{L}^\dagger_n \right)$$

2. Auxiliary Observable Variational Principle

- We find that the Lindblad equation can be reformulated as a variational principle, with action

$$S[\widehat{D}, \widehat{A}] = \hbar \mathrm{Tr} \widehat{D}(t_f) \widehat{A}_f - \int_{t_i}^{t_f} dt \mathrm{Tr} \widehat{A} \left\{ \hbar \frac{d\widehat{D}}{dt} + i[\widehat{H}, \widehat{D}] - \hbar \sum_n \left(\frac{1}{2}\{\widehat{L}^\dagger_n \widehat{L}_n, \widehat{D}\} - \widehat{L}_n \widehat{D} \widehat{L}^\dagger_n \right) \right\}$$

- By the variation with respect to the density matrix and the auxiliary observable, one can recover the Lindblad equation and its adjoint equation

3. Generalized Time-Dependent Hartree-Fock-Bogoliubov Equation

- Under Hartree-Fock-Bogoliubov approximation (density matrix is an infinite-mode fermionic Gaussian, observable is a general quadratic observable), the resulting variational equation of motion is

$$\boxed{i\hbar\dot{\mathcal{R}} = [\mathcal{H}_R, \mathcal{R}] + i\hbar\{\mathcal{H}_I, \mathcal{R}\} + i\hbar\mathcal{J}}$$

one-body density matrix

$$\mathcal{R} = \begin{pmatrix} \rho & \kappa \\ \kappa^\dagger & 1-\rho^* \end{pmatrix}$$

$$\rho_{\mathbf{k},\mathbf{k}'} \equiv \rho_{\mathbf{k},\uparrow,\mathbf{k}',\uparrow} = \rho_{\mathbf{k},\downarrow,\mathbf{k}',\downarrow} = \mathrm{Tr}(\widehat{D}(t) c^\dagger_{\mathbf{k}',\uparrow} c_{\mathbf{k},\uparrow})$$

$$\kappa_{\mathbf{k},\mathbf{k}'} \equiv \kappa_{\mathbf{k},\downarrow,\mathbf{k}',\uparrow} = \kappa_{\mathbf{k},\uparrow,\mathbf{k}',\downarrow} = \mathrm{Tr}(\widehat{D}(t) c_{\mathbf{k}',\uparrow} c_{\mathbf{k},\downarrow})$$

Hartree-Fock-Bogoliubov Hamiltonian

$$\mathcal{H}_{R/I} = \begin{pmatrix} h_{R/I} & \Delta_{R/I} \\ \Delta^\dagger_{R/I} & -h^*_{R/I} \end{pmatrix} \ (h_{R/I})_{\mathbf{k},\mathbf{k}'} = \frac{\partial \langle \widehat{H}_{R/I} \rangle}{\partial \rho_{\mathbf{k}',\uparrow,\mathbf{k},\uparrow}},\ (\Delta_{R/I})_{\mathbf{k},\mathbf{k}'} = \frac{\partial \langle \widehat{H}_{R/I} \rangle}{\partial \kappa^*_{\mathbf{k},\uparrow,\mathbf{k}',\downarrow}}$$

"effective quantum jump"

$$\mathcal{J} = \begin{pmatrix} 0 & 2\kappa h^*_I - 2\rho\Delta_I \\ 2h^*_I\kappa^\dagger - 2\Delta^\dagger_I\rho & \{h^*_I, 1-2\rho^*\} - 2\Delta^\dagger_I\kappa - 2\kappa^\dagger\Delta_I \end{pmatrix}$$

- One observed that a "non-Hermitian Hartree-Fock" approximation is enough for the normal phase system that does not hold any pair coherence:

$$\boxed{i\hbar\dot{\rho} = [h_R, \rho] + i\hbar\{h_I, \rho\}}$$

4. Wigner Transform and Semi-classical Approximation

- We can rewrite the generalized time-dependent Hartree-Fock-Bogoliubov equation into phase space by the Wigner transform

$$O(\mathbf{r}, \mathbf{p}) = \int d^3 s e^{-i\mathbf{p}\cdot\mathbf{s}/\hbar} O_{\mathbf{r}+\frac{\mathbf{s}}{2}, \mathbf{r}-\frac{\mathbf{s}}{2}}$$

- The Wigner transform turns all matrix multiplications into Moyal products. To systematically do a semi-classical approximation, we expand all Moyal products up to the first order of the reduced Planck constant

$$A(\mathbf{r},\mathbf{p}) \star B(\mathbf{r},\mathbf{p}) = AB + \frac{i\hbar}{2}\{A, B\}_c + \mathcal{O}(\hbar^2)$$

- The Poisson bracket

$$\{A, B\}_c \equiv \sum_{i=x,y,z} \left(\frac{\partial A}{\partial r_i}\frac{\partial B}{\partial p_i} - \frac{\partial A}{\partial p_i}\frac{\partial B}{\partial r_i} \right)$$

5. Inelastic Quantum Boltzmann Equation for Bogoliubov Quasi-particles

- We find that under the semi-classical approximation, the time-dependent Hartree-Fock-Bogoliubov equation supports Bogoliubov quasi-particles with infinite lifetime, whose phase space distribution evolves under an inelastic quantum Boltzmann equation

$$\boxed{\dot{\nu} = \{E, \nu\}_c + h_I(2\nu - 1 + h^{\mathrm{ev}}_R/E^{\mathrm{ev}})}$$

- For the homogeneous system in the BCS limit, the equation can be explicitly written as

Quasi-particle distribution evolution — *Physical particle distribution evolution*

$$\boxed{\dot{\nu} = \left(-2\gamma + 2g_I \int \frac{d^3p}{(2\pi\hbar)^3}\rho\right) \frac{2\nu^2 - 2\nu + \rho}{2\nu - 1}} \quad \textit{with} \quad \boxed{\int d^3p(\dot{\rho} - 2h_I\rho) = 0}$$

Results

We find an analytical solution of the homogeneous system case, which reads

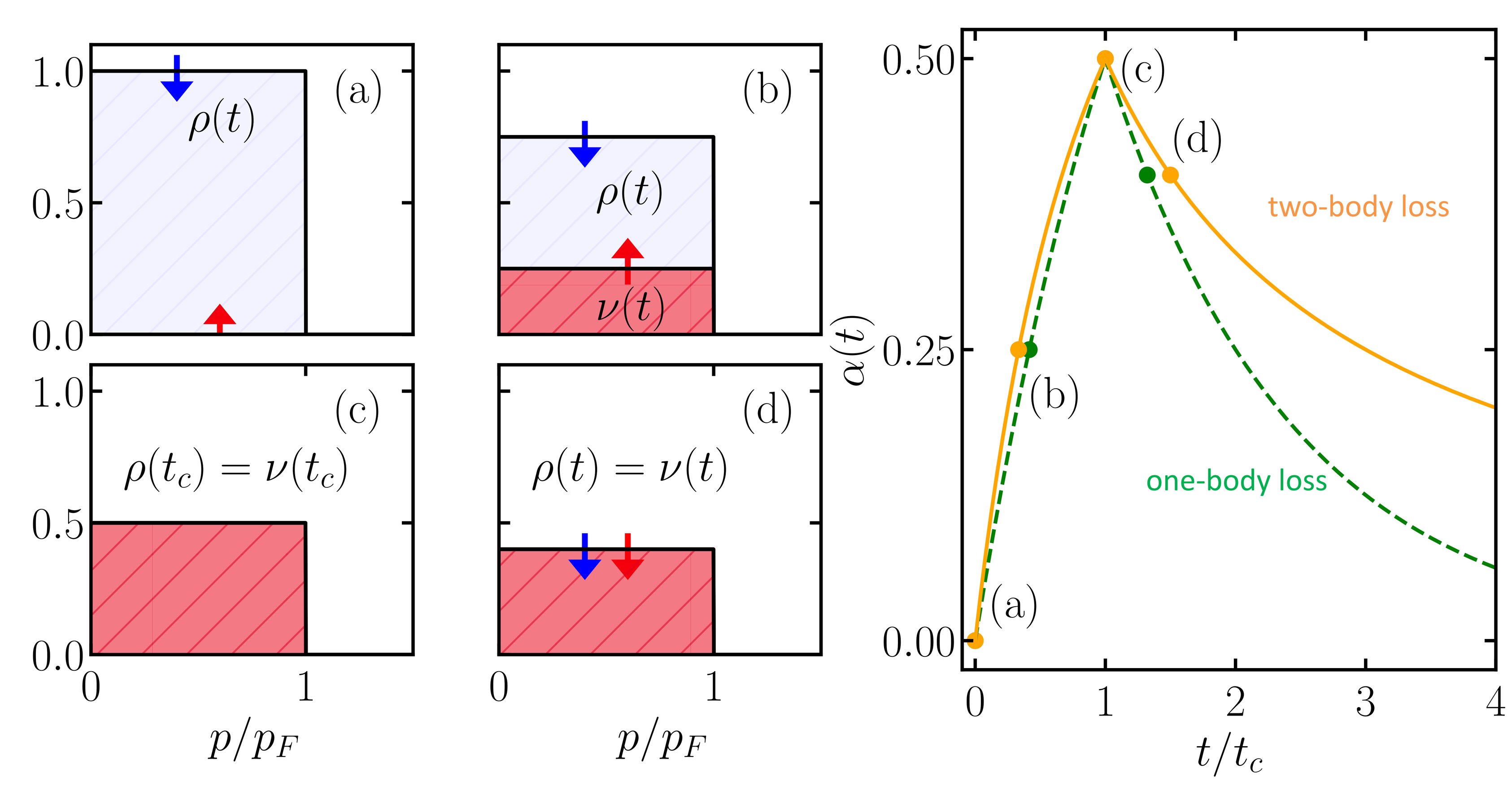
$$\nu = \frac{\alpha(t)}{p_F}\theta(1 - p/p_F), \quad \rho = \frac{1-\alpha(t)}{p_F}\theta(1 - p/p_F)$$

where (if there is only one- or two-body loss)

$$\alpha(t; g_I = 0) = \frac{1}{2} - \frac{1}{2}e^{-2\gamma t}|e^{2\gamma t} - 2|, \quad \alpha(t; \gamma = 0) = \frac{1}{2} - \frac{|g_I^2 k_F^6 t^2 - 9\pi^4|}{2(g_I k_F^3 t + 3\pi^2)^2}.$$

One sees a **non-analytical behavior at a "critical time":**

$$t_c(g_I = 0) = \ln(2)/2\gamma, \quad t_c(\gamma = 0) = -3\pi^2/g_I k_F^3.$$



Four stages of the dissipative evolution

(a) System in the ground state: there are no excitations, thus no quasi-particles

(b) Losses create quasi-particles (hole-excitation)

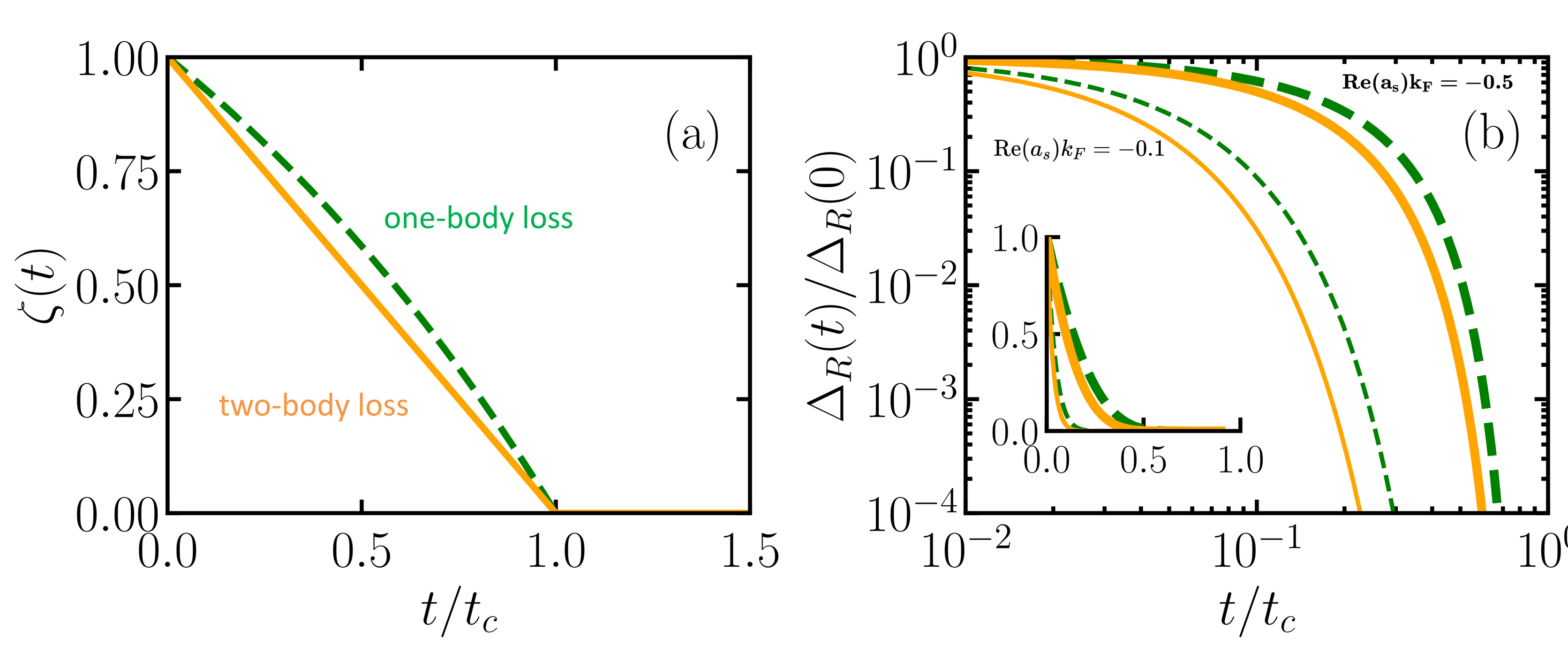
(c) Quasi-particle distribution coincides with which of physical particles

(d) The quasi-particle becomes the same as physical particles: superfluidity is completely diminished.

We calculate how the superfluid fraction and superfluid gap evolve for further verification:

$$\zeta(t) = \begin{cases} [1 - 2\alpha(t)]/[1 - \alpha(t)] & t < t_c, \\ 0 & t \geq t_c. \end{cases}, \qquad \Delta_R(t) = \begin{cases} \dfrac{8E_F}{e^2}\exp\left\{\dfrac{\pi}{2k_F \mathrm{Re}(a_s)[1 - 2\alpha(t)]}\right\} & t < t_c \\ 0 & t \geq t_c, \end{cases}$$

It is observed that the **superfluid fraction is discontinuous in the first-order time derivative**, and the **superfluid gap is smooth but non-analytical.**



Acknowledgements and References

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