Temperature-dependent contact of weakly interacting single-component Fermi gases and loss rate of degenerate polar molecules

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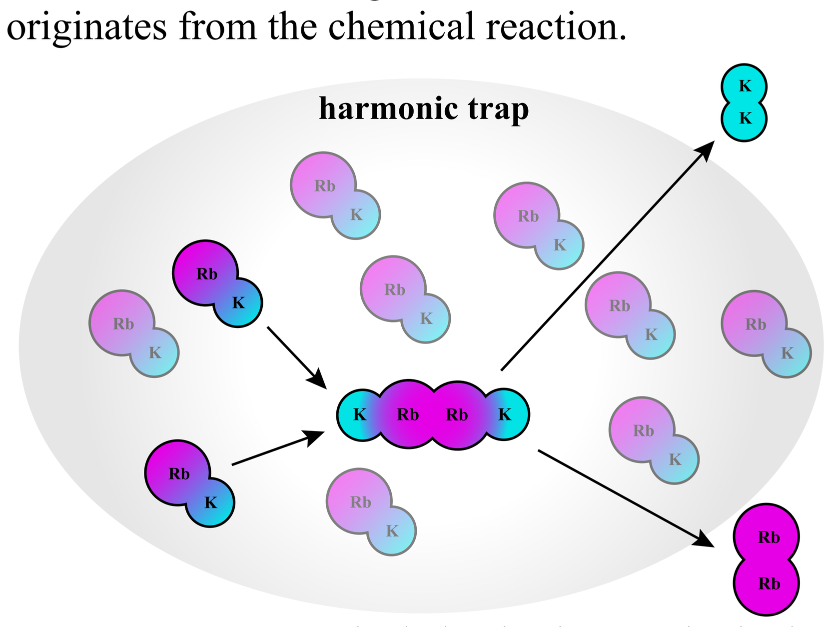
Abstract

Ground-state KRb polar molecules have been cooled to quantum degeneracy. The inelastic collision between two molecules, due to chemical reactions, gives rise to loss over time. Below the Fermi temperature, a surprising suppression of the loss rate was observed experimentally. An explanation is currently lacking, even for the seemingly "simple" model of a normal-phase dilute weakly-interacting single-component Fermi gas. Typically, more than one microscopic parameter is needed to describe interactions between identical fermions. Nevertheless, here, we identify a single relevant thermodynamic intensive microscopic parameter, the *p*-wave scattering volume, and its corresponding thermodynamic extensive variable, the contact, and develop a unified statistical mechanics framework. Using the framework, we obtain the temperature-dependent contact, and, from it, the normal-phase loss rate. Our work reproduces the measured loss rate of the ultracold reactive KRb molecular gas for all experimentally accessible temperatures without adjustable parameters.

Two-body loss of degenerate molecular Fermi gas

In 2019, experimentalists realized molecular $^{40}K^{87}Rb$ gases with temperatures T as low as $T/T_F \approx 0.3$, where T_F denotes the Fermi temperature.[1] Different from a degenerate atomic gas, the degenerate molecular Fermi gas is unstable due to its two-body loss, which originates from the chemical reaction.



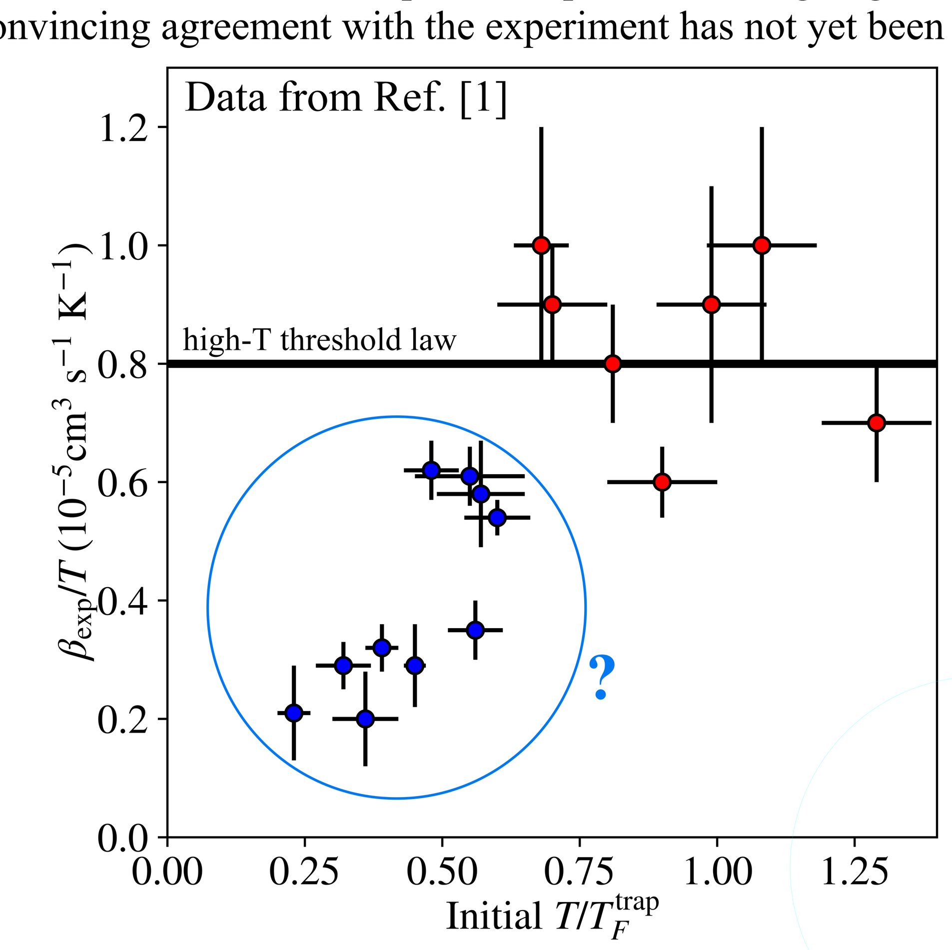
In a homogeneous system, the behavior is quantitatively described by the two-body loss coefficient β:

$$\frac{dn}{dt} = -\beta n^2.$$

Since in a harmonically trapped system, a global density is not naturally defined, experiments conventionally, fit the loss coefficient β_{exp} using

$$\frac{dn^{\text{trap}}}{dt} = -\left[\beta_{\text{exp}} + \frac{3}{2}\frac{1}{T_{\text{exp}} n^{\text{trap}}}\frac{dT_{\text{exp}}}{dt}\right](n^{\text{trap}})^2,$$

where n^{trap} and T_{exp} are the *in-situ* average density and temperature measured from the ballistic expansion process, respectively.
At high temperatures, β_{exp}/T is expected to be constant based on the Bethe-Wigner threshold law for *p*-wave collisions. Interestingly, a suppression in the deep degenerate regime was observed.
Several works [2–3] attempted to explain this intriguing behavior but convincing agreement with the experiment has not yet been yielded.



Relation between two-body loss and *p*-wave contact

By integrating out the products as well as intermediate four-body complexes, a single-component Fermi gas with two-body loss can be described by a non-Hermitian Hamiltonian with a complex interaction

$$\Re(\hat{H}) = \int d^3r \psi^\dagger(\vec{r})\psi(\vec{r}) + \frac{1}{2}\int d^3r d^3r' \Re(U)\psi^\dagger(\vec{r})\psi^\dagger(\vec{r}')\psi(\vec{r}')\psi(\vec{r})$$

$$\Im(\hat{H}) = \frac{1}{2}\int d^3r d^3r' \Im(U)\psi^\dagger(\vec{r})\psi^\dagger(\vec{r}')\psi(\vec{r}')\psi(\vec{r})$$

The Hamiltonian suggests the Lindblad jump operator

$$\hat{L}_{|\mathbf{r}-\mathbf{r}'|} = \sqrt{\Im[U(|\mathbf{r}-\mathbf{r}'|)]}\Psi(\mathbf{r}')\Psi(\mathbf{r})$$

From the Lindblad master equation: $\hbar\frac{\partial\langle\hat{N}\rangle}{\partial t} = 4\Im\langle\hat{H}\rangle$.
In the weakly interacting limit, the *p*-wave phase shift

$$\delta_p = \arctan\left(-v_p k^3 + \frac{v_p^2}{R}k^5\right)$$ almost disappear, thus $v_p \to 0$,

$\langle\hat{H}\rangle \to \left\langle\frac{\partial\hat{H}}{\partial v_p}\right\rangle v_p \to \frac{\partial F}{\partial v_p}v_p$. For the homogeneous system:

$$\beta = -\frac{6\hbar}{mnN}C_v\frac{\Im(v_p)}{[\Re(v_p)]^2} \quad \text{and} \quad \left.\frac{\partial F}{\partial v_p^{-1}}\right|_R = -\frac{3\hbar^2}{2m}C_v$$

where C_v is the *p*-wave contact conjugate to the scattering volume v_p.

Homogeneous two-body loss coefficient

At high temperatures, the thermodynamics of the system can be described by the virial expansion up to second order, which yields:

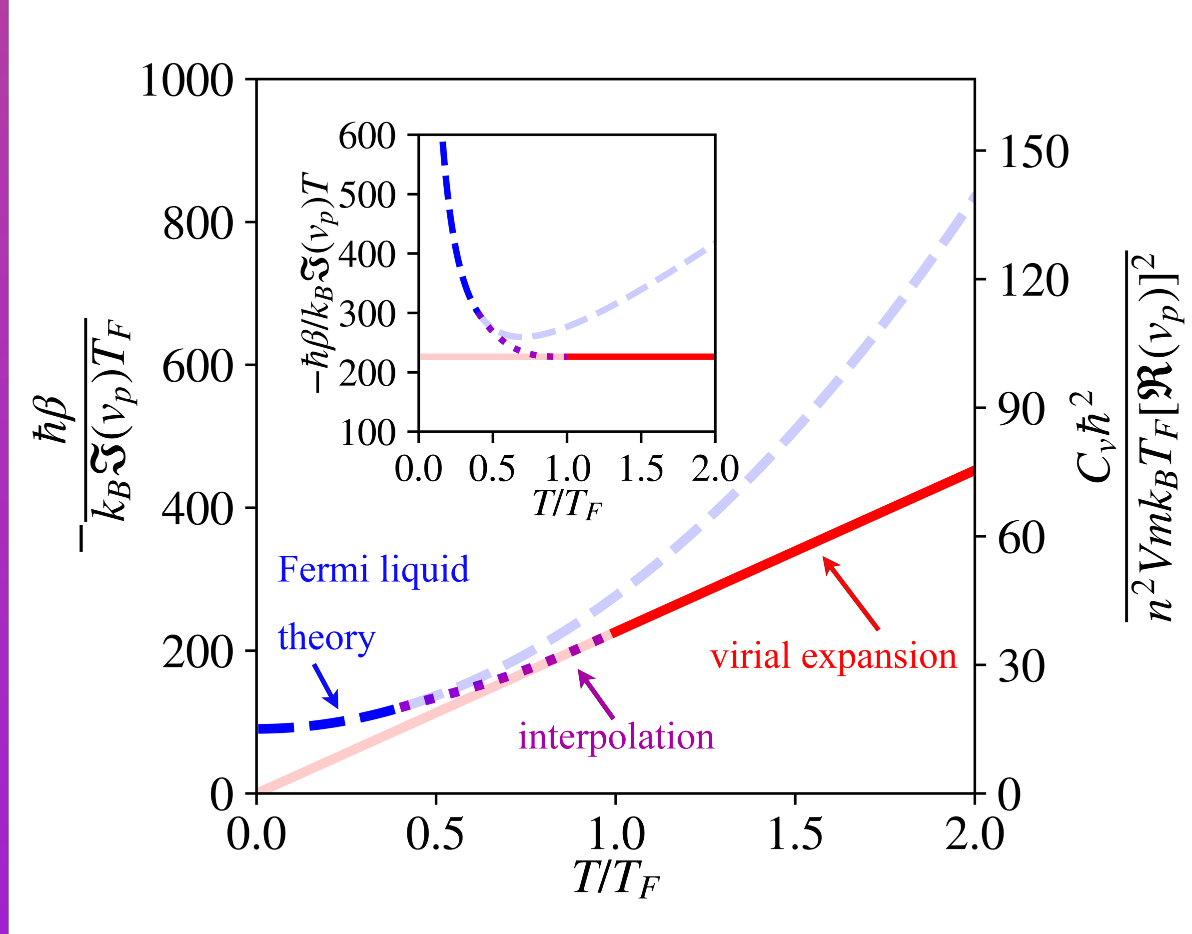
$$C_v = \frac{12\pi m k_B T n^2 V(\Re(v_p))^2}{\hbar^2}, \quad \beta = -\frac{72\pi k_B}{\hbar}T\Im(v_p).$$

In the deep degenerate regime, the system is described by *p*-wave Fermi liquid theory [4], yielding

$$C_v = \frac{12\times 6^{2/3}\pi^{7/3}n^{8/3}V(\Re(v_p))^2}{5} + \frac{2^{1/3}\pi^{5/3}m^2k_B^2T^2n^{4/3}V(\Re(v_p))^2}{3^{2/3}\hbar^4}$$

$$\beta = -\frac{144\pi k_B}{5\hbar}T_F\Im(v_p) - \frac{6\pi^3 k_B}{\hbar}\frac{T^2}{T_F}\Im(v_p)$$

The behavior in the intermediate regime can be obtained by performing a smooth interpolation between the two limits, because there is no phase transition in this temperature regime.



Local density approximation: two-body loss coefficient of harmonically trapped cloud

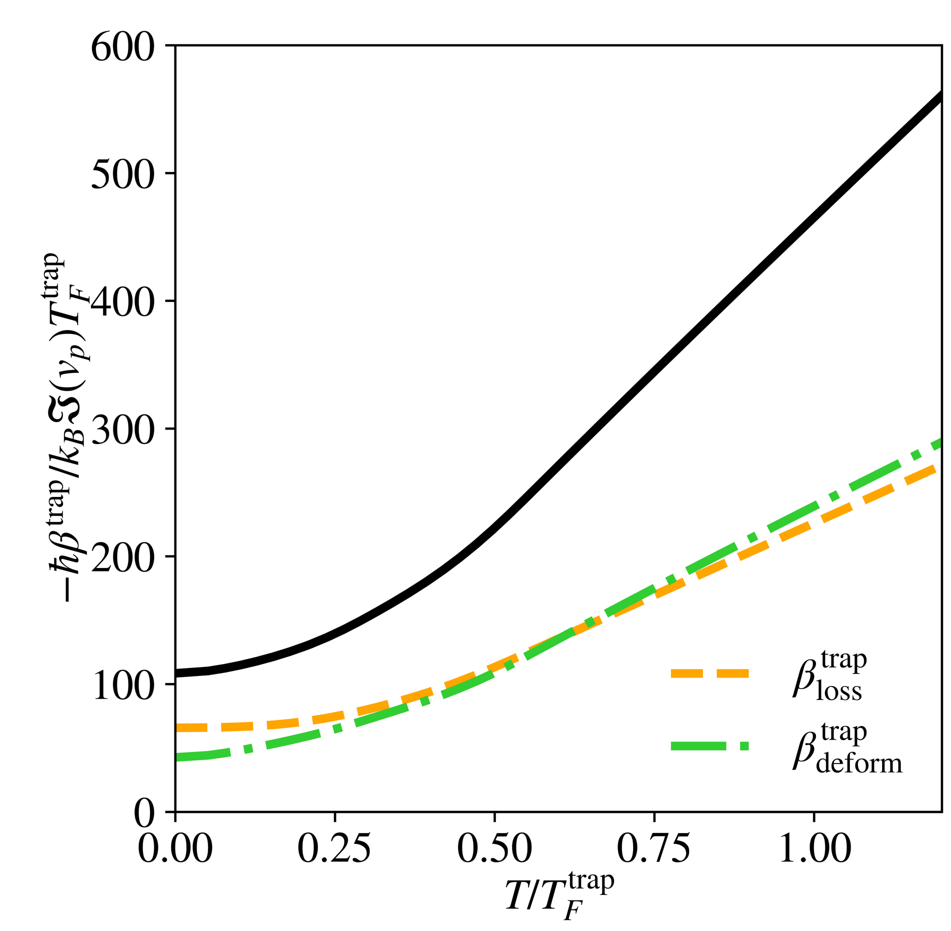
In analogy to the homogeneous system and following the usual convention of the experiment [1], we define the *total* loss coefficient β^{trap} for the harmonically trapped system to be

$$\frac{dn^{\text{trap}}}{dt} = -\beta^{\text{trap}}(n^{\text{trap}})^2, \quad n^{\text{trap}} = N/V^{\text{trap}}, \quad V^{\text{trap}} = N^2\left[\int d^3r(n^{\text{trap}})^2\right]^{-1}.$$

Because the "volume" V^{trap} is no longer fixed, β^{trap} separates naturally into two contributionss: $\beta^{\text{trap}} = \beta^{\text{trap}}_{\text{loss}} + \beta^{\text{trap}}_{\text{deform}}$

$$\beta^{\text{trap}}_{\text{loss}} = -\frac{V^{\text{trap}}}{N^2}\frac{dN}{dt} = \frac{\int d^3r\beta(\mathbf{r})(n^{\text{trap}})^2}{\int d^3r(n^{\text{trap}})^2},$$

$$\beta^{\text{trap}}_{\text{deform}} = \frac{1}{N}\frac{dV^{\text{trap}}}{dt} = \frac{2\left(\int d^3\mathbf{r}n(\mathbf{r})\right)\left(\int d^3\mathbf{r}\beta(\mathbf{r})[n(\mathbf{r})]^3\right)}{\left(\int d^3\mathbf{r}[n(\mathbf{r})]^2\right)^2} - \frac{2\int d^3\mathbf{r}\beta(\mathbf{r})[n(\mathbf{r})]^2}{\int d^3\mathbf{r}[n(\mathbf{r})]^2}$$



Fit the experiment

1) $\beta(T) = \beta^{\text{trap}}_{\text{loss}} = \beta_{\text{exp}}$ at $T \gg T_F^{\text{trap}}$, but not in the degenerate regime. Using this fact, we fit $\Im(v_p) = -(136a_0)^3$ from a previous high-temperature experiment [5].

2) For the convenience of data processing, most experiments assume a classical profile of the cloud

$$n^{\text{trap}} \to n^{\text{trap}}_{\text{exp}} = \frac{N\bar{\omega}^3}{8}\left(\frac{m}{\pi k_B T_{\text{exp}}}\right)^{3/2},$$

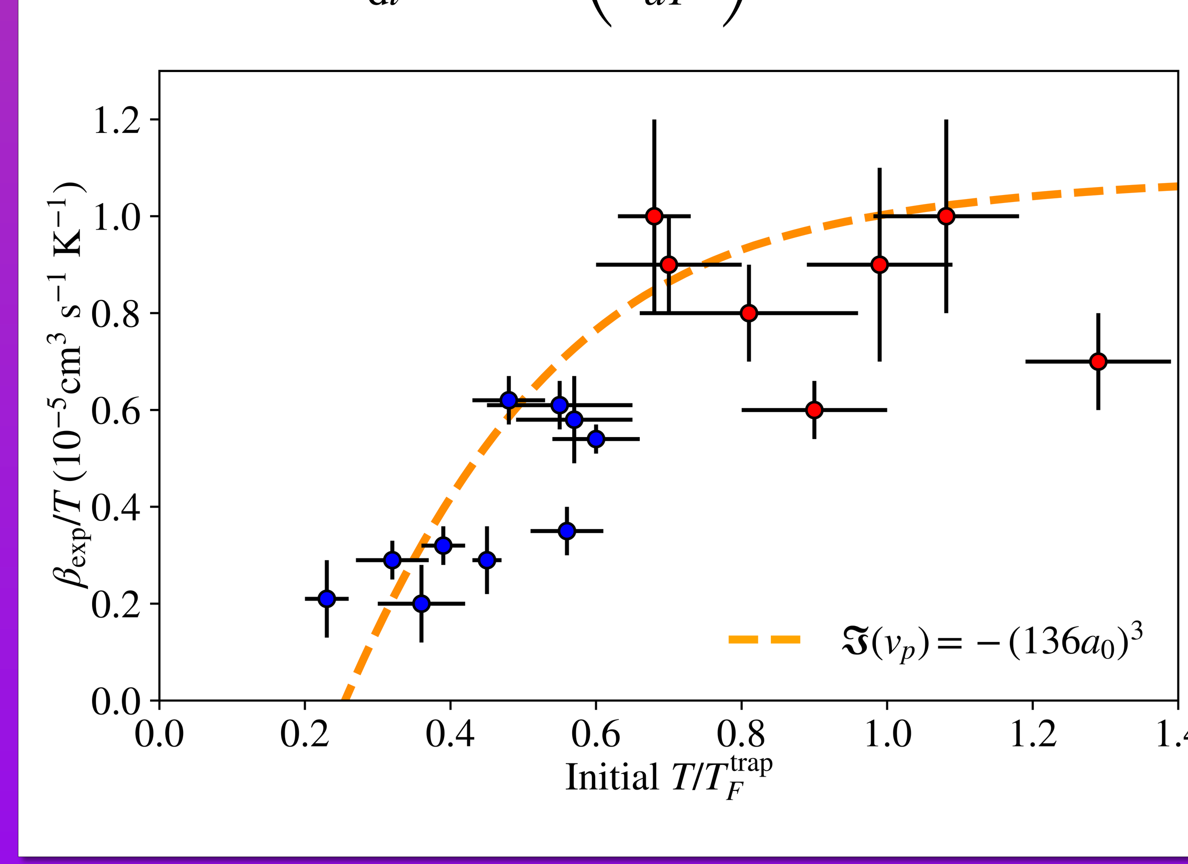
which constraints the form of $\beta^{\text{trap}}_{\text{deform}}$ to $\frac{3}{2}\frac{1}{T_{\text{exp}}n^{\text{trap}}_{\text{exp}}}\frac{dT_{\text{exp}}}{dt}$.

We can determine β_{exp} using β^{trap} and $\beta^{\text{trap}}_{\text{deform}}$:

$$\beta_{\text{exp}} = \left[\frac{(n^{\text{trap}})^2}{(n^{\text{trap}}_{\text{exp}})^2}\frac{dn^{\text{trap}}_{\text{exp}}}{dn^{\text{trap}}}\right]\beta^{\text{trap}} - \frac{3}{2}\left(\frac{1}{T_{\text{exp}}}\frac{dT_{\text{exp}}}{dT}\right)\left(\frac{1}{n^{\text{trap}}_{\text{exp}}}\frac{dT}{dt}\right),$$

where the $\beta^{\text{trap}}_{\text{deform}}$ is proportional to the temperature change("anti-evaporation") reads

$$\frac{dT}{dt} = \beta^{\text{trap}}_{\text{deform}}\left(\frac{dn^{\text{trap}}}{dT}\right)^{-1}(n^{\text{trap}})^2$$



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