Abnormal Superfluid Fraction of Harmonically Trapped Few-Fermion Systems

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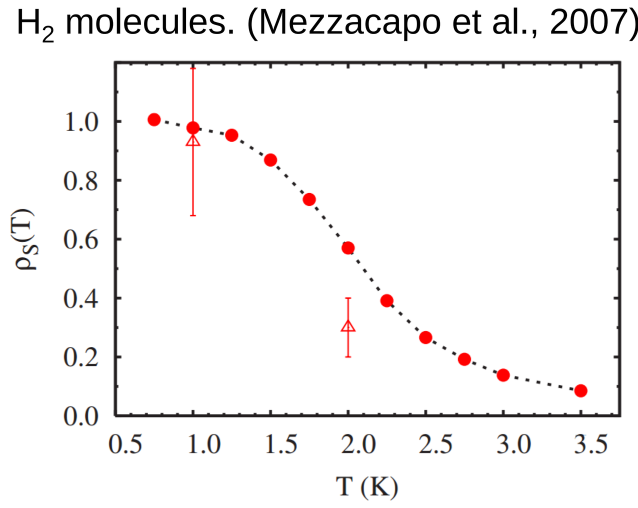
Abstract

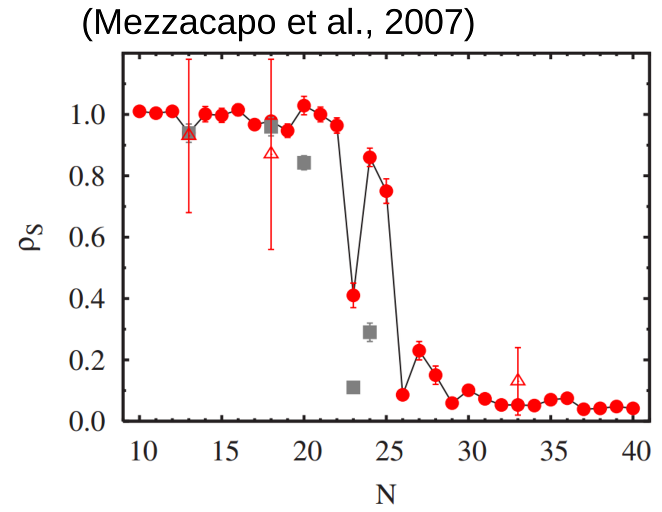
Superfluidity is a fascinating phenomenon that, at the macroscopic scale, leads to dissipationless flow and the emergence of vortices. While these macroscopic manifestations of superfluidity are well described by theories that have their origin in Landau's two-fluid model, our microscopic understanding of superfluidity is far from complete. Using analytical and numerical *ab initio* approaches, this work determines the superfluid fraction and local superfluid density of small harmonically trapped two-component Fermi gases as a function of the interaction strength and temperature. At low temperature, we find that the superfluid fraction is, in certain regions of the parameter space, negative. This counterintuitive finding is traced back to the symmetry of the system's ground state wave function, which gives rise to a diverging quantum moment of inertia I_q. Analogous abnormal behavior of I_q has been observed in even-odd nuclei at low temperature. Our predictions can be tested in modern cold atom experiments.

Earlier Work on Microscopic Superfluidity

Temperature dependence of the superfluid fraction for a clusters of 18 p-H_2 molecules. (Mezzacapo et al., 2007)



Superfluid fraction of p-H_2 clusters versus cluster size N at T=1K. (Mezzacapo et al., 2007)

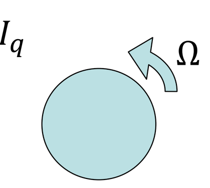


Moment of Inertia and Superfluid Fraction

We consider N equal-mass atoms described by the Hamiltonian H in a spherically symmetric harmonic trap. The system Hamiltonian under a small rotation Ω about the z-axis can, in the rotating frame, be expressed as $H_{rot} = H - \Omega L_z$.
The superfluid fraction n_s is defined as $n_s = 1 - I_q/I_c$. This has been wildly applied to studies of bulk helium (macroscopic system) and clusters (microscopic system) and provided good explanation.

The quantum moment of inertia I_q

$$I_q = \frac{\partial \langle L_z \rangle_{\text{th}}}{\partial \Omega}\Big|_{\Omega=0}$$

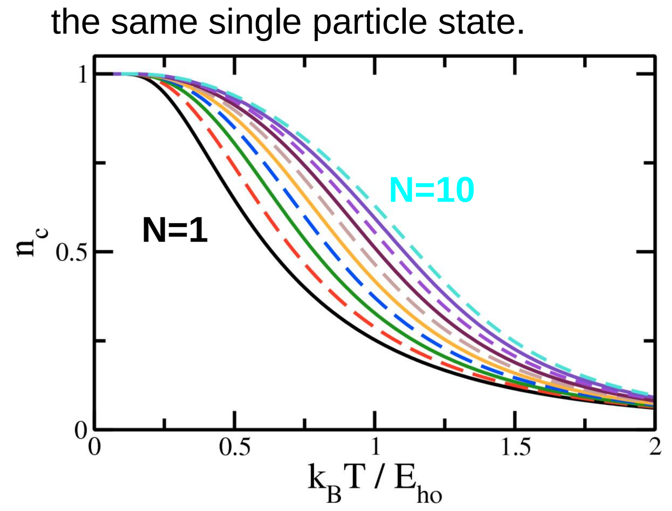


The classical moment of inertia I_c

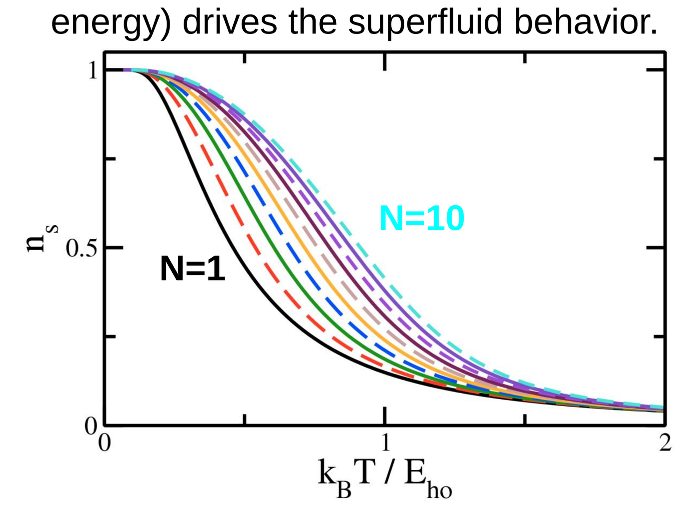
$$I_c = \langle m \sum_n r_{n,\perp}^2 \rangle_{\text{th}}$$

Microscopic Description of Trapped Non-Interacting Bose System

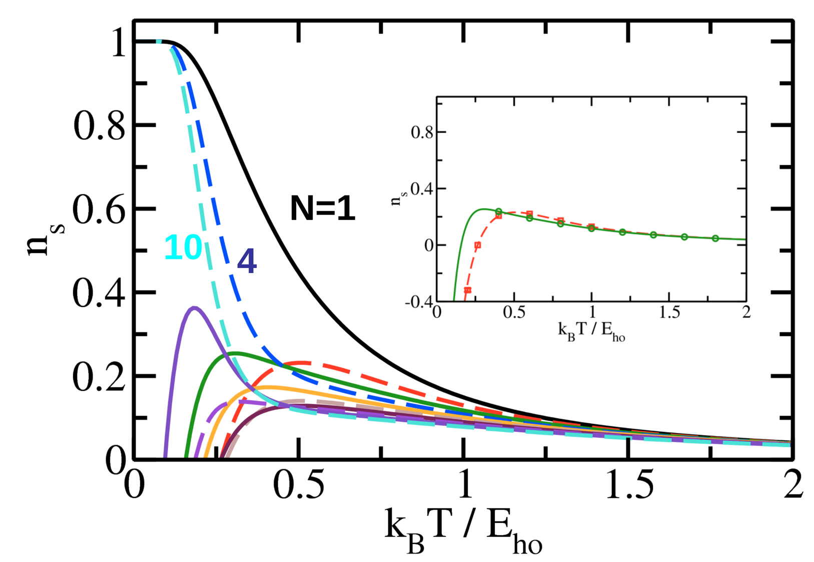
The condensate fraction is defined as the largest occupation number of the natural orbitals. In the zero temperature limit all particles occupy the same single particle state.



Trapped few-boson systems become superfluid at low temperature.
The energy gap between the ground state and the first excited state (i.e., the trap energy) drives the superfluid behavior.



Single-Component Non-Interacting Fermi Gas



Superfluid fraction n_s as a function of temperature T. Symbols: Path integral Monte Carlo using area estimator.

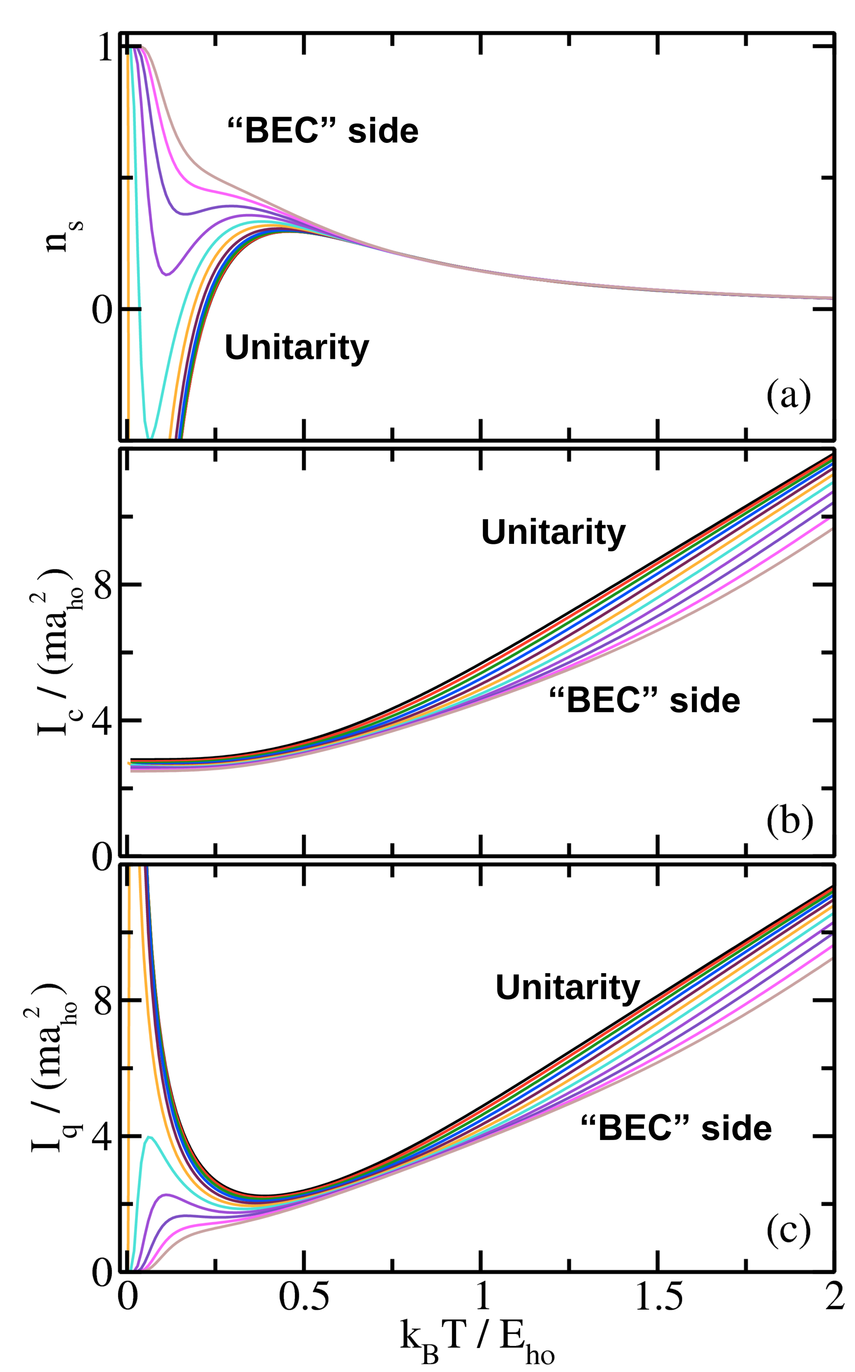
One can show that $I_q = \hbar^2 \langle M^2 \rangle_{\text{th}} / (k_B T)$.
In the zero temperature limit,
$I_q = 0, n_s = 1$
if system ground state $L = 0$ (close shell).
$I_q = \infty, n_s = -\infty$
if system ground state $L = 1$ (open shell).

Negative superfluid fraction indicates I_q larger than I_c, i.e., system rotates faster than expected.

Two-Component Fermi Gas

We consider two-component Fermi gases consisting of N_1 spin-up and N_2 spin-down particles with short-range interspecies interactions [denoted as (N_1, N_2) systems].
For the (2,1) and (2,2) systems at low temperature, we employ the zero-range model potential and calculate the energy spectrum.
For the (2,2) system at high temperature, we employ a short-range Gaussian potential in the path integral Monte Carlo calculations. The finite range effect is small.

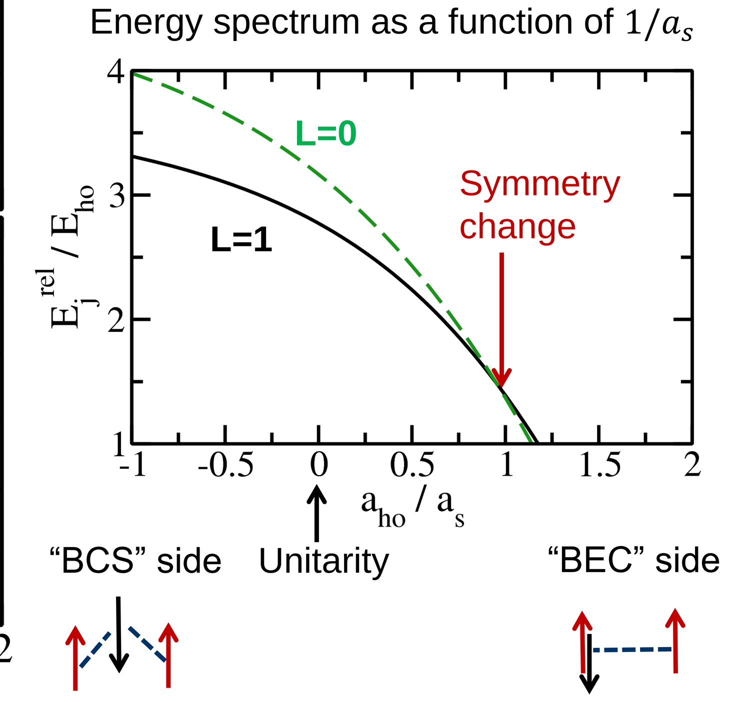
Two-Component Fermi Gas: (2,1) System



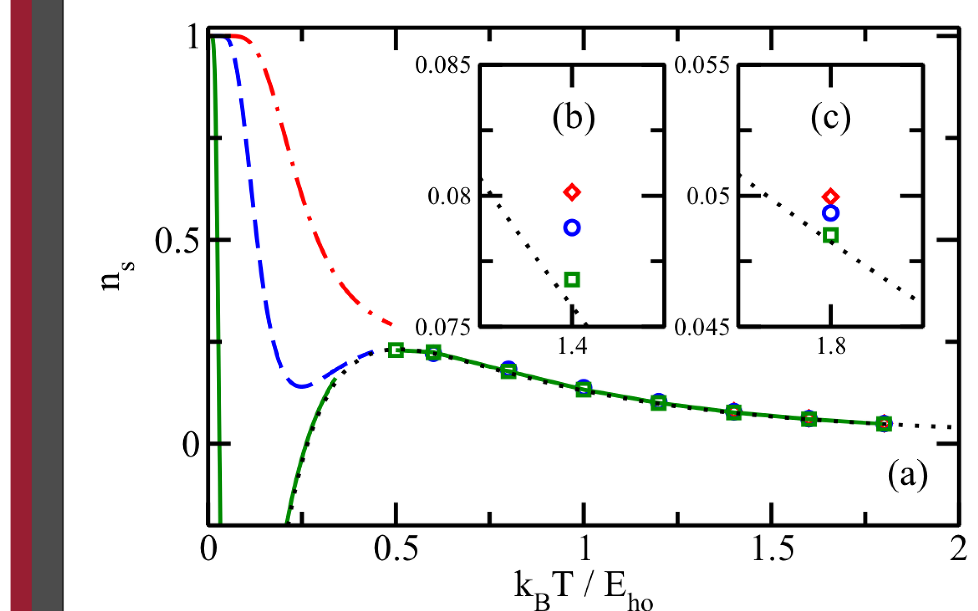
Properties of the interacting trapped (2,1) system as a function of temperature.
(a) The lines from bottom to top show n_s for a_{ho}/a_s=0, 0.2, …, 2.
(b)/(c) The lines from top to bottom show I_c and I_q, respectively, for a_{ho}/a_s = 0, 0.2, …, 2.

"Quantum phase transition like" feature around a_{ho}/a_s=1.

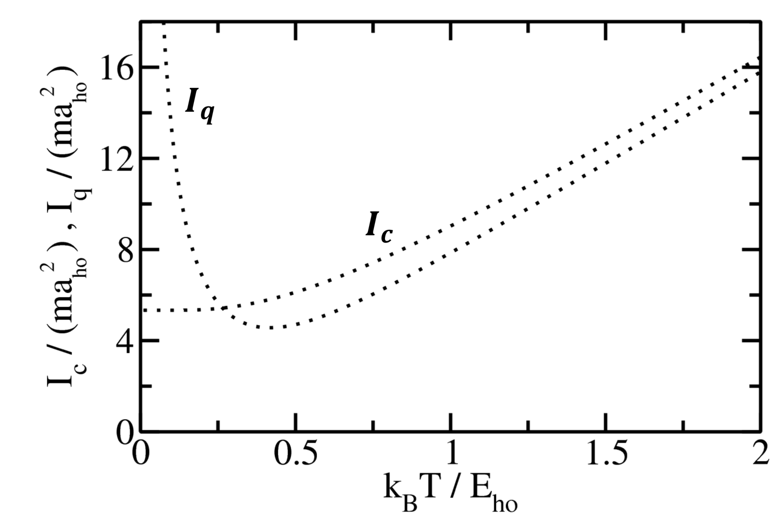
Energy spectrum as a function of $1/a_s$

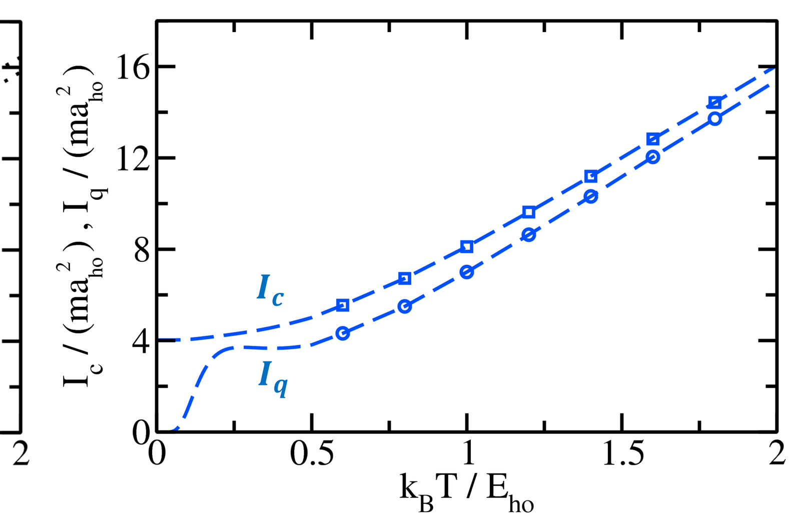


Two-Component Fermi Gas: (2,2) System



Properties of the trapped (2,2) system.
(a) The dotted, solid, dashed, and dash-dotted lines show n_s as a function of temperature for a_s/a_{ho}= 0, −0.2, −1, and ∞, respectively.
The squares, circles, and diamonds show n_s obtained by the PIMC approach for a_s/a_{ho}= 0, −0.2, −1, and ∞, respectively.
(b) and (c) show blowups of the high-temperature region.

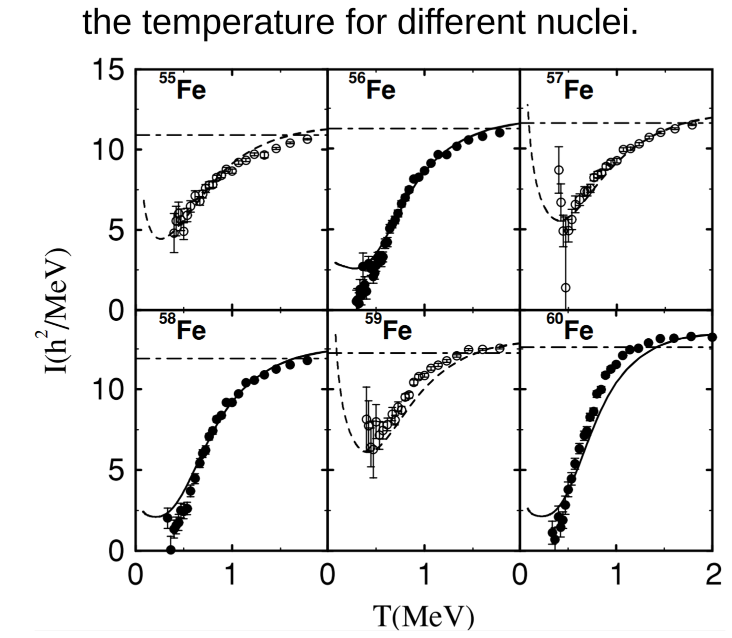




Conclusion

We observe a microscopic "quantum phase transition like" feature driven by the interspecies s-wave interaction for few-fermion systems in a harmonic trap. In certain parameter regimes, we find that the superfluid fraction is negative.

Even/odd effect in nuclei (Alhassid et al., 2005). Moment of inertia as a function of the temperature for different nuclei.



Comparison of superfluidity and superconductivity

Superfluid fraction n_s	negative susceptibility $-\chi$
Rotation Ω	magnetic field $\mu_0 H$
Angular momentum L	magnetic flux BS
$(1-n_s) = \frac{\partial L}{\partial \Omega} / I_c$	$(1+\chi) = \frac{\partial B}{\partial \mu_0 H}$
$n_s < 0$ rotate faster than expected.	$\chi > 0$ paramagnetic.
$n_s = 0$ normal fluid.	$\chi = 0$ nonmagnetic.
$n_s > 0$ rotate slower than expected.	$\chi < 0$ diamagnetic.
$n_s = 1$ perfect superfluid.	$\chi = -1$ perfect superconductor.

References

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