Emergent periodic and quasiperiodic lattices on surfaces of synthetic Hall tori and synthetic Hall cylinders

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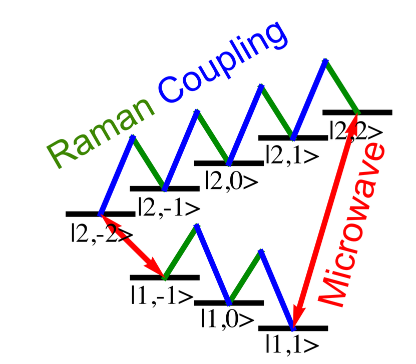
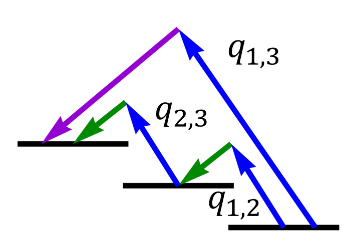
Introduction & Brief Summary

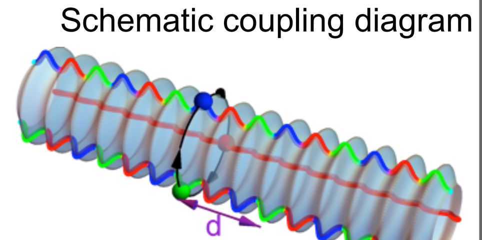
- Synthetic dimension can be exploited to engineer Hamiltonians that are not possible in real space: e.g. torus or cylinder whose surface is threaded by a uniform magnetic field.
- We propose realizations of synthetic Hall tori and cylinders using Bose Einstein condensate whose internal states are cyclically coupled and study the properties.
- Lattice forms if there is a net momentum transfer ($q_{1,2}+q_{2,3}\neq q_{1,3}$)
  - If momentum transfers are commensurate => Ordinary lattice
    - Exact crossing in energy spectrum protected by nonsymmorphic symmetry
    - Bloch oscillation has a period that is multiples of the period defined by the Hamiltonian
  - If incommensurate => Quasiperiodic lattice
- Gluing two tori (cylinders) with incommensurate momentum transfer => localized ground state.
  - Changing coupling strength results in localization-delocalization transition
  - Robust against weak perturbation of interaction.



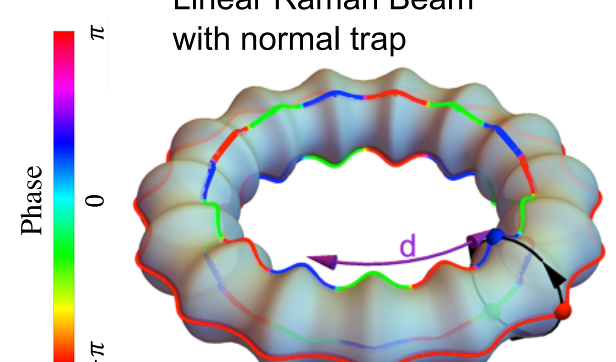
Rb hyperfine spin splitting



Schematic coupling diagram



Linear Raman Beam with normal trap



Laguerre Gaussian Beam with Ring trap

Schematic figure of cylinder and torus. The density oscillation is depicted as the fluctuation of the radius of the torus or cylinder.

Hamiltonian

The Hamiltonian reads

Kinetic Energy + Detuning

$$H=\sum_{j=1}^{M}|\psi^j(x)\rangle\left(-\frac{\hbar^2}{2m_0}\partial_x^2+\epsilon_j\right)\langle\psi^j(x)|$$
$$+\sum_{j=1}^{M}\left(\Omega_{j,j+1}e^{iq_{j,j+1}x}|\psi^{j+1}(x)\rangle\langle\psi^j(x)|+h.c.\right)$$

Cyclic coupling:
$\Omega_{j,j+1}$: Coupling Strength
$q_{j,j+1}$: momentum transfer

- M internal states (M+1=1).

Commensurate Case

$q_{j,j+1}=n_jq_L$, where n_j are integers.

q_L is the size of Brillouin zone (BZ). $H(x)=H(x+\frac{2\pi}{q_L})$ $d\equiv\frac{2\pi}{q_L}$ the lattice spacing

1. Lattice translational symmetry
2. Nonsymmorphic symmetry. Translate a fraction of d followed by a change of phase for each internal state. $x\rightarrow x+\frac{2\pi}{Q},\ |\psi^{j>1}\rangle\rightarrow e^{-i\frac{2\pi}{Q}\sum_{j'=1}^{j-1}q_{j',j'+1}}|\psi^j\rangle$, where $Q\equiv\sum_{j=1}^{M}q_{j,j+1}$

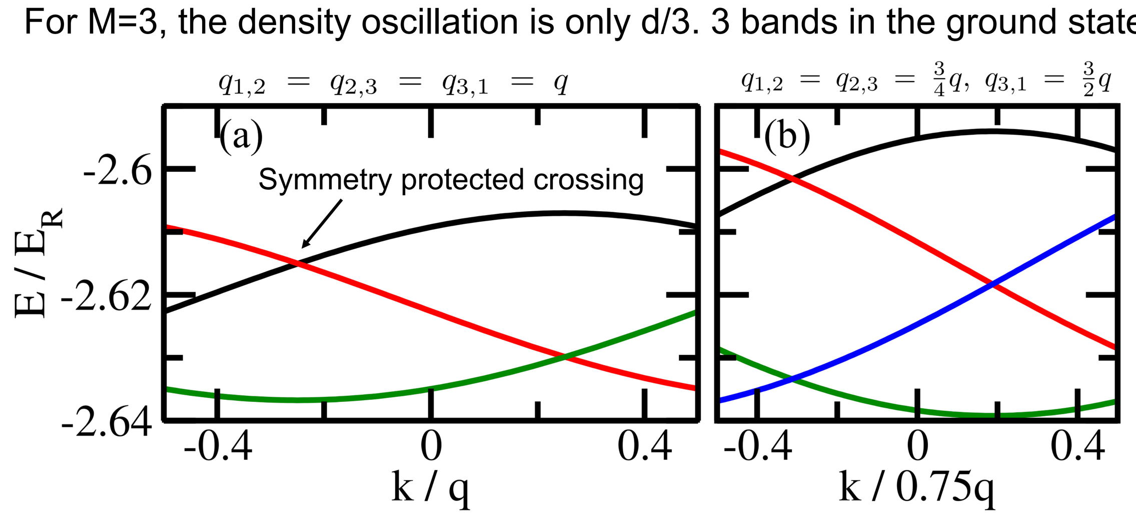
   Eigenvalue: $c_s=e^{i(\frac{kd}{n}+\frac{2s\pi}{n})}$ $s=1,2,...n-1,n.$

We consider system with $n_j=1$

Eigen state for quasi-momentum k: $\vec{\psi}_k(x)=e^{i(k+sq)x}(u_k^1(x),u_k^2(x),u_k^3(x))^T$

Periodic part: $u_k^j(x)=e^{i(j-1)qx}\sum_{l=-\infty}^{\infty}c_l^j(k)e^{ilQx}$

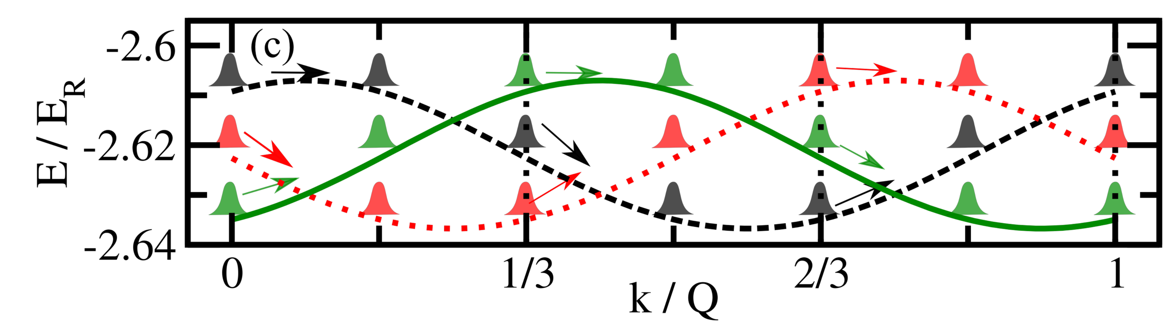
For M=3, the density oscillation is only d/3. 3 bands in the ground state cluster.



Band structures in the first BZ. Reduction of the size of the first BZ leads to increase of the number of bands.

$\Omega_{1,2}=1.2E_R$, $\Omega_{2,3}=1.8E_R$, and $\Omega_{3,1}=1.5E_R$ $E_R=\hbar^2Q^2/2m_0$ is the recoil energy defined by Q.

Braiding and Period Tripling in the Bloch Oscillation

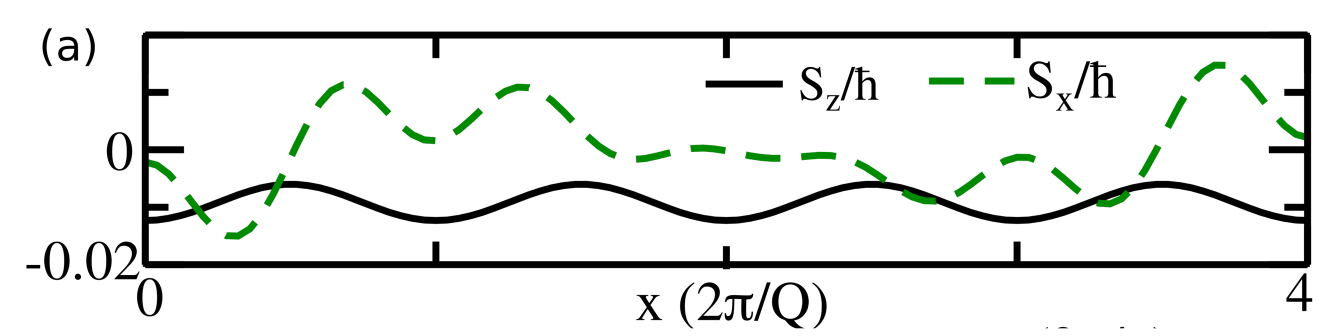


Bloch oscillation starts after applying a weak constant force.
The period of the Bloch oscillation is Q=3q.
The 6 possible permutation form the braiding group.

Incommensurate case: Quasiperiodic lattice

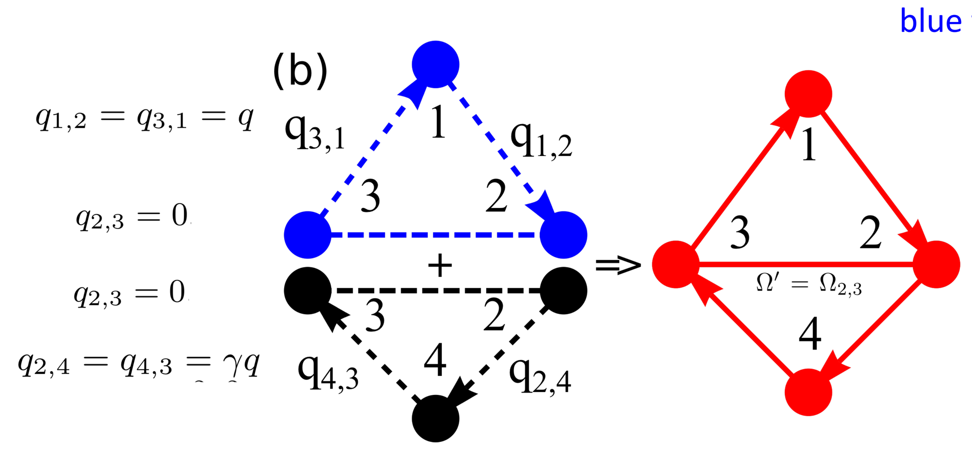
e.g., $q_{1,2}:q_{2,3}:...:q_{M,1}=1:1:...:\gamma$, where $\gamma=\frac{\sqrt{5}-1}{2}$

The Hamiltonian is no longer periodic. However, the density oscillation has a period of $\frac{2\pi}{Q}$.

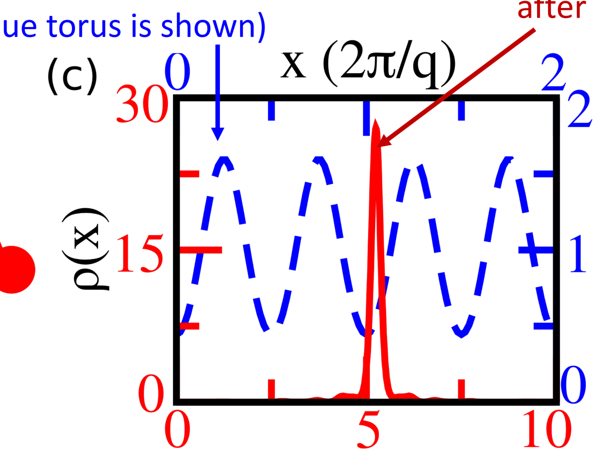


Spin polarization along the x and z direction as a function of x.

Localization by gluing two tori with incommensurate momentum transfer



Schematic of the cross sections of two tori when they are glued together.



Total density of the ground eigenstate at k=0 before and after the gluing.

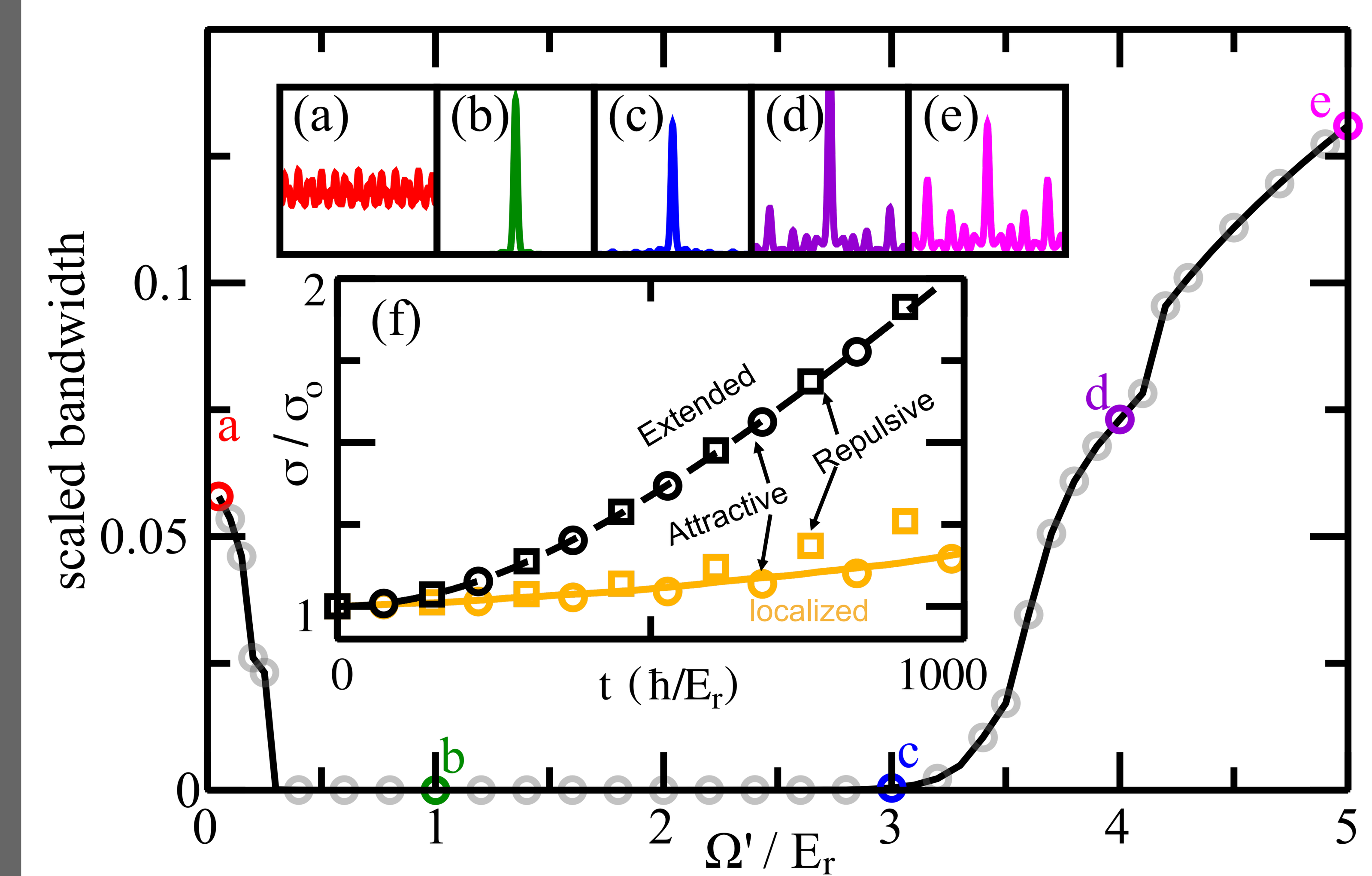
All $\Omega_{j,j'}$ are $2E_r$, where $E_r=\hbar^2q^2/2m_0$ is the recoil energy defined by q. We use $\gamma_8=13/21$ to approximate γ

Fibonacci series $\{a_\alpha\}$: 1,1,2,3,5,8,13,21…

Gluing two tori with incommensurate momentum transfer result in localized states. Highly excited states are still extended (mobility edge exists in the energy spectrum).

It has been shown that the ground band width scales with inverse lattice spacing squared for extended states, and decays much faster for localized states [1].

Fix all other coupling strength and vary Ω' (hopping between state 2 and 3).
Vanishing Ω': quasiperiodic lattice; extended states.
Large Ω': state 2 and 3 dominate; still extended states.
Localization is only possible in intermediate Ω'.



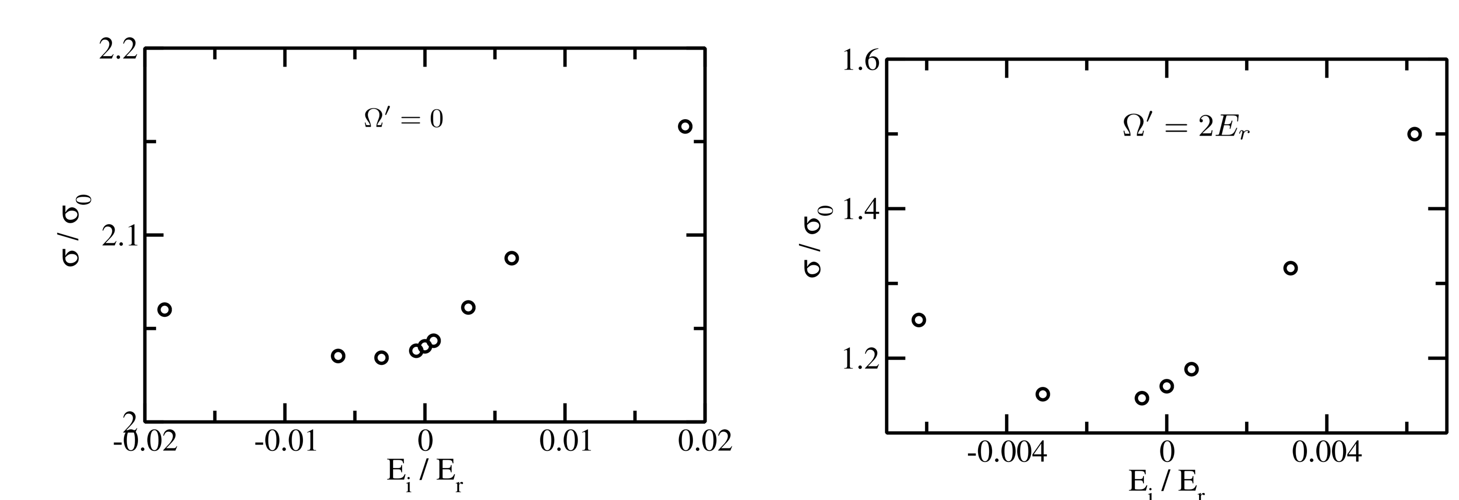
Bandwidth multiplied by $(a_\alpha/E_r)^2$ as a function of Ω' for $\alpha=12$. The inset (a),…,(e) show the corresponding ground state density at $\Omega'/E_r=0.05,1,3,4,5$, respectively with $\alpha=8$. (f) The width of the Gaussian wave packet as a function of time The solid orange (black dashed) curve represents non-interacting results for $\Omega'=2E_r$ ($\Omega'=0$). Squares and circles represent results for $\int g\rho_i^2\mathrm{d}x=\pm0.003E_r$, respectively, where ρ_i is the density for the initial state.

Wave packet expansion: GP calculation with interaction.

Consider a gaussian wave packet (each spin component has equal amplitude and phase). Time evolve using time-dependent Gross-Pitaevskii equation.

$$i\hbar\frac{\partial\vec{\psi}(x)}{\partial t}=(\hat{H}+g\rho)\vec{\psi}(x)$$ g the interaction strength. ρ the density.

Regardless of weak interactions, wave packet expands slowly (fast) in the localized (extended) phase. The slow expansion in the localized phase is due to the finite overlap of the initial state to the extended state.



Density spread as a function of initial interaction energy E_i after evolving time $1000\hbar/E_r$.

References

•[1] R. B. Diener, G. A. Georgakis, J. Zhong, M. Raizen, and Q. Niu, Transition between extended and localized states in a one-dimensional incommensurate optical lat- tice, Phys. Rev. A 64, 033416 (2001).

[2] Yangqian Yan, Shao-Liang Zhang, Sayan Choudhury, and Qi Zhou arXiv:1810.12331